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Multicriterion Preliminary Design of a Tetrahedral Truss Platform

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An efficient method is presented for multicriterion preliminary design and demonstrated for a tetrahedral truss platform. The method requires minimal analysis effort and permits rapid estimation of optimized truss behavior. A 14-m-diam, three-ring truss platform represents a reflector support structure for space-based science spacecraft. Truss members are grouped by truss ring and position. Design variables are the cross-sectional area of all members in a group, and are integer multiples of the minimum member area. Nonstructural mass represents the node and joint hardware used for assembly of the truss structure. Taguchi methods are used to identify key points in the set of Pareto-optimal designs, which are the trusses with maximum frequency, minimum mass, and maximum frequency-to-mass ratio. Low-order polynomial curve fits through the key points are used to approximate the behavior of the full set of Pareto-optimal designs. The resulting Pareto-optimal design curve is used to predict frequency and mass for optimized trusses. Performance improvements are plotted in frequency–mass (criterion) space and compared with results for uniform trusses. Application of constraints to the criteria and sensitivity to constraint variation are demonstrated.

Nomenclature

A	= normalized truss member area
f	= truss fundamental frequency, Hz
f/m	= truss frequency-to-mass ratio, Hz/kg
m	= truss total mass, kg
$\alpha, \beta, \gamma, \delta$	= coefficients for frequency curve fit
$\varepsilon, \zeta, \eta, \theta$	= coefficients for mass curve fit

Introduction

DESIGN of aerospace systems inevitably requires simultaneous satisfaction of many different, and usually conflicting, criteria.[1] Various techniques have been developed for solution of multicriterion optimization problems.[2] In one common method, a single scalar objective function is formed from the weighted sum of the criteria of interest and used for optimization analyses. By varying the relative weights assigned to the criteria, the set of Pareto-optimal configurations may be found.[3] A configuration is called a Pareto optimum if no improvement may be made in one criterion without simultaneous degradation of at least one of the other criteria.[4] In this study, an efficient method is presented for estimating the behavior of the Pareto-optimal designs for preliminary design studies, and demonstrated for a tetrahedral truss platform.

Several proposed Earth-science and deep-space astrophysics spacecraft (Fig. 1) require large, high-precision truss platforms on the order of 10 m in diameter or larger[5] to support faceted reflector surfaces. A lattice truss, which has inherently high natural frequencies and low mass, is a likely candidate for the primary mirror support structure. A 14-m-diam, three-ring tetrahedral truss platform, shown in Fig. 2a, is representative of the reflector support structures described previously. Previous design studies[6] of this structure used natural frequency as the sole optimization criterion.

In the present study, the two criteria of interest are the natural frequency and mass of the truss. Trusses that have been optimized for a combination of high frequency and low mass form the set of Pareto-optimal designs. A search through the entire design space for the set of Pareto-optimal designs would be both tedious and time-consuming. Instead, a simplified process is presented here in which certain key designs from the Pareto-optimal set are determined, and the behavior of the remaining Pareto-optimal designs is inferred from the behavior of the key designs.

The three key designs chosen have maximum frequency, minimum mass, and maximum frequency-to-mass ratio. Approximate solutions for these designs are determined with Taguchi methods, and results are plotted in frequency–mass (criterion) space. The characteristics of the full set of Pareto-optimal designs are modeled with low-order polynomial curves that are fitted through the three key design points in criterion space. Finally, a study is performed to illustrate how the Pareto-optimal design curve can be used for preliminary design. Uniform and optimized truss frequency and mass are determined for nominal constraints, along with performance sensitivity for changes in the constraints.

Truss Geometry and Analysis Model

The tetrahedral truss configuration evaluated in this study is composed of 315 truss members, each 2 m in length. A perspective view of the truss platform and a repeating cell are shown in Fig. 2a. The truss, which has a diameter of 14 m across corners and a depth of 1.63 m, is shown subdivided into three circumferential truss rings in Fig. 2b, and partitioned into upper surface, core, and lower surfaces

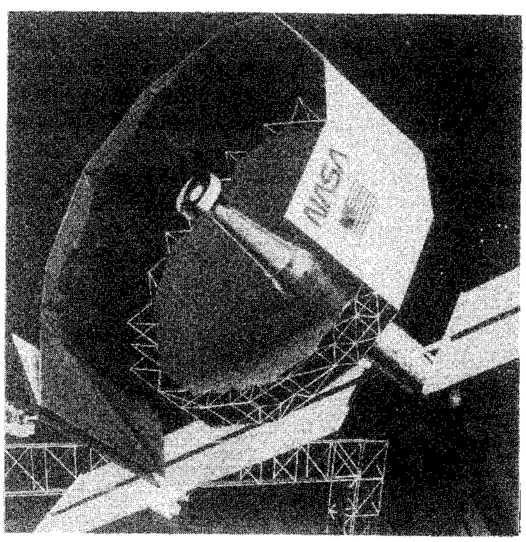

Fig. 1 20-m-diam high-precision reflector.

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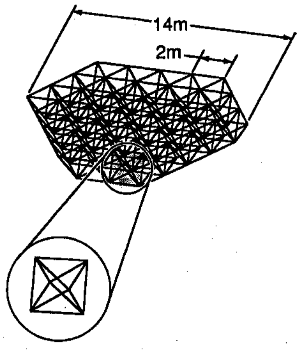
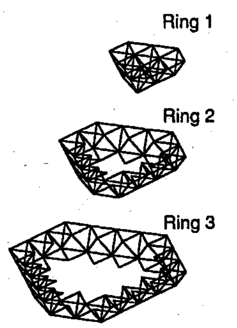
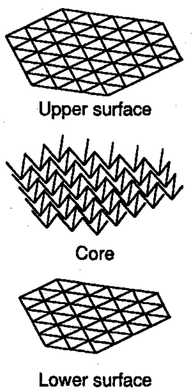


a) Perspective view with unit cell



b) Subdivided into truss rings



c) Subdivided into upper surface, core, and lower surface

Fig. 2 Three-ring tetrahedral truss platform.

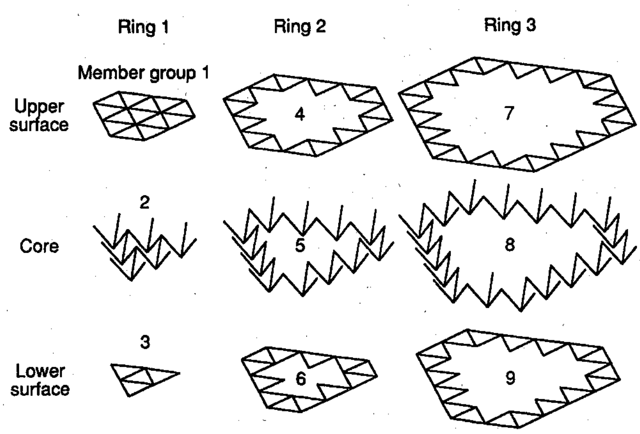


Fig. 3 Member groups for three-ring truss.

in Fig. 2c. The truss subdivisions in Figs. 2b and 2c are used to define the nine member groups shown in Fig. 3. The design variables for this problem are the truss member cross-sectional areas. Every truss member in a given group has the same cross-sectional area of either 1, 3, or 5 times the minimum area. Since there are three possible values for each of the nine design variables, there are $3^9 =$ 19,683 discrete truss configurations in the entire design space.

A commercial structural analysis code[7] is used to construct a linear finite element model that has pinned-end axial stiffness elements. The truss members are assumed to be composed of a composite tube with a fixed outer diameter of 3.18×10^{-2} m and a joint at each end, which attaches to a truss node. The minimum cross-sectional area of the truss members is 6.45×10^{-5} m^2, which is assumed to be sufficient to satisfy minimum gauge and member buckling requirements. The truss members are assumed to have an elastic modulus of 1.23×10^{11} Pa and mass density of 1348 kg/m^3, nominal properties for a high-performance graphite–epoxy material. Nonstructural mass is included to represent the node and joint hardware used to assemble the truss structure. Each truss node is represented by a 0.39-kg point mass, and each truss joint is represented by a 0.21-kg point mass. A total of 162.41 kg of nonstructural mass is distributed among the 84 truss nodes. An eigenvalue analysis is performed to compute the lowest flexible-body frequency of the truss with free–free boundary conditions. The mode shape associated with the fundamental frequency is asymmetric, anticlastic bending of the truss.[6]

Optimization Criteria

A high truss natural frequency f is typically desirable to maintain adequate separation between structural and attitude-control-system frequencies. Also, high natural frequencies result in lower dynamic amplitudes and faster damping of transient disturbances, so stringent spacecraft pointing and surface accuracy requirements for science missions may be met. Thus, it is particularly important to maintain high natural frequency for trusses used in precision reflectors, where the panel support structure must be both stiff and accurate.[8]

The truss mass m is another criterion for structural optimization problems. Low mass is necessary because of the extremely high cost of transportation to orbit, with launch costs for current systems on the order of 10,000 dollars per kilogram.[9] In addition, high masses are often reflected in higher costs for additional material, fabrication, and processing. In this study, both frequency and mass are criteria used in determining the Pareto-optimal designs.

Pareto-Optimal Design Curve

In this section, characteristics and construction of a Pareto-optimal design curve are discussed for optimization criteria of frequency and mass. The Pareto-optimal design curve is used to approximate the behavior of the actual set of Pareto-optimal designs in criterion space. As the design parameters are varied, the frequency and mass of the structure are assumed to vary within certain limits. These limits can be graphically determined by plotting frequency and mass for all possible designs. The shaded region in the criterion-space plot of Fig. 4 represents such a plot of all possible configurations for a given design problem. Because designs that combine high frequency and low mass are desired, the upper-left-hand boundary of this region of feasible designs is identified as the Pareto-optimal boundary and is shown in the figure as a dark solid line. Thus, the Pareto-optimal design curve is an approximation to this maximum-frequency, minimum-mass boundary of the region of feasible designs. Any deviation from this boundary into the region of feasible designs will result in a decrease in frequency or an increase in mass, or both.

Three key points—the designs with maximum frequency, minimum mass, and maximum frequency-to-mass ratio—are identified on the Pareto-optimal boundary in Fig. 4. The maximum-frequency design and the minimum-mass design are important because they are the endpoints of the Pareto-optimal boundary. The maximum-frequency design is the upper limit of the Pareto-optimal boundary, and the slope through this point must be zero (no feasible designs exist above this point). Similarly, the minimum-mass design defines

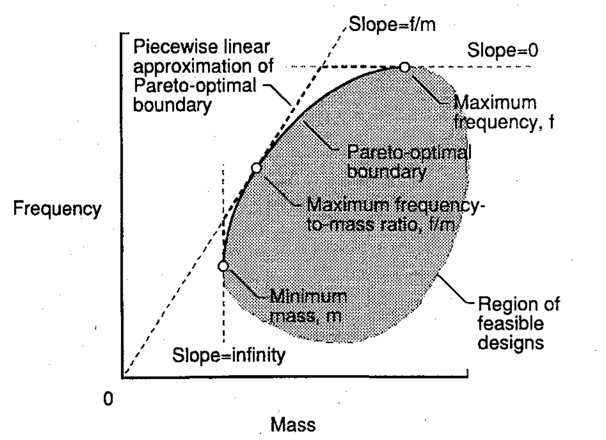


Fig. 4 Criterion-space plot of feasible designs and Pareto-optimal boundary.

the left-hand limit of the Pareto-optimal boundary, and the slope through this point must be infinite (no designs exist to the left of this point). Finally, the design with maximum frequency-to-mass ratio is assumed to be the upper-left-hand limit of the Pareto-optimal boundary. The slope of a line through that design and the origin is equal to f/m, its frequency-to-mass ratio.

A simple, nonconservative approximation of the Pareto-optimal boundary consists of the three bounding lines described above. This piecewise linear design curve is shown in Fig. 4 as a dark dashed line. More conservative approximations of the design curve can be constructed from low-order polynomial curve fits through the key points and their slopes described previously. The accuracy of such curve fits depends on the behavior of the actual Pareto-optimal boundary, which is assumed to be both smooth and continuous in this study.

Computation of Key Truss Designs

To determine the Pareto-optimal design curve for the truss example, Taguchi methods[10] are used to identify the truss designs with maximum frequency, minimum mass, and maximum frequency-to-mass ratio. Taguchi methods, an unconstrained optimization technique, use orthogonal arrays from design-of-experiments theory to reduce the effort necessary to locate optimized designs. For any pair of columns in an orthogonal array, every possible combination of array elements occurs an equal number of times. Also, each column is linearly independent of every other column. The orthogonal array size is chosen, based on the number of design variables and the number of values that each design variable may have. For this problem, an orthogonal array with 27 rows is selected using the program of Ref. 11.

The design variables, which are the cross-sectional areas of the nine member groups in Fig. 3, are assigned to the nine columns of the orthogonal array in Table 1. Since allowable values for the design variables are 1, 3, and 5 times the minimum area, the array elements in Table 1 are defined as the normalized member area A, or the ratio of the member area to the minimum area. In this paper, truss configurations are presented in the form (123 456 789), where each value in parentheses is the normalized area for member groups 1–9.

As noted previously, there are $3^9 = 19{,}683$ possible combinations of design variables and areas, which are collectively termed the full-factorial set. Each of the 19,683 trusses is assumed to be a feasible design. Thus, the values of the design variables must be selected with manufacturing and logistical considerations in mind. For example, use of a composite material for the truss members restricts the area to discrete values because of the integer number of laminae in the tube wall.

In the present study, 27 analyses are required to determine an optimized truss configuration. Thus, each row of the array in Table 1 represents an analysis case that must be evaluated. In each case, the design variables are set to the values indicated in the row of the orthogonal array, and the frequency, mass, and frequency-to-mass ratio are computed using the vibrational analysis described previously. The values of these objective functions are shown in Table 1 for each of the 27 cases. For example, the truss configuration in analysis case 25 has normalized areas of (551 513 135), and frequency, mass, and frequency-to-mass ratio of 44.20 Hz, 336.17 kg, and 0.1315 Hz/kg. The frequency, mass, and frequency-to-mass ratio data from Table 1 are reduced in Table 2a. Each table entry is the average of the objective function values for a given truss member group (1–9) at a given normalized member area (1, 3, or 5). For example, the 35.80-Hz average frequency from Table 2a for member group 1 and normalized area 1 is the average of the computed frequencies for all cases in which member group 1 has a normalized area of 1 (cases 1–9 from Table 1).

The optimized truss design is determined by setting the cross-sectional area of each of the nine design variables to the value (1, 3, or 5) that yields the best results (shown in boldface in Table 2) of the three allowable values. Thus, for the maximum-frequency truss design, member group 1 will have a normalized member area of 5, since the corresponding average frequency of 38.83 Hz from Table 2a is higher than the other frequencies in that column. The predicted maximum-frequency truss configuration, shown in Table 2a, has normalized member areas of (555 535 135). This design resembles a tapered plate, where the thickness at the perimeter is less than the thickness at the center. The minimum-mass truss configuration, shown in Table 2b, is one where every member group has the lowest possible normalized area, 1. Since the mass of a member is proportional to its area, this result makes physical sense. The predicted truss configuration with the maximum frequency-to-mass ratio, shown in Table 2c, has normalized areas of (533 315 111). This truss is similar to a tapered honeycomb sandwich, with stiff face sheets in truss rings 1 and 2, minimum-mass face sheets in ring 3, and a low-mass core throughout the truss.

Table 1 Orthogonal array and structural analysis results for Taguchi analysis

	Normalized area A for truss member group											
Case	1	2	3	4	5	6	7	8	9	f, Hz	m, kg	f/m, Hz/kg
1	1	1	1	1	1	1	1	1	1	29.93	217.20	0.1378
2	1	1	3	5	5	5	5	5	5	39.94	404.01	0.0989
3	1	1	5	3	3	3	3	3	3	38.13	315.30	0.1209
4	1	3	1	5	5	5	3	3	3	39.61	351.83	0.1126
5	1	3	3	3	3	3	1	1	1	36.19	263.12	0.1375
6	1	3	5	1	1	1	5	5	5	31.36	340.35	0.0921
7	1	5	1	3	3	3	5	5	5	36.84	376.87	0.0978
8	1	5	3	1	1	1	3	3	3	32.03	288.16	0.1112
9	1	5	5	5	5	5	1	1	1	38.20	309.04	0.1236
10	3	1	1	5	3	1	5	3	1	31.58	323.65	0.0976
11	3	1	3	3	1	5	3	1	5	39.31	316.34	0.1243
12	3	1	5	1	5	3	1	5	3	38.39	321.56	0.1194
13	3	3	1	3	1	5	1	5	3	42.72	320.52	0.1333
14	3	3	3	1	5	3	5	3	1	34.92	328.87	0.1062
15	3	3	5	5	3	1	3	1	5	36.97	330.95	0.1117
16	3	5	1	1	5	3	3	1	5	35.46	326.78	0.1085
17	3	5	3	5	3	1	1	5	3	40.00	335.13	0.1194
18	3	5	5	3	1	5	5	3	1	38.74	337.21	0.1149
19	5	1	1	3	5	1	3	5	1	32.48	329.91	0.0985
20	5	1	3	1	3	5	1	3	5	41.58	322.60	0.1289
21	5	1	5	5	1	3	5	1	3	39.06	334.08	0.1169
22	5	3	1	1	3	5	5	1	3	36.67	329.91	0.1112
23	5	3	3	5	1	3	3	5	1	39.71	338.26	0.1174
24	5	3	5	3	5	1	1	3	5	41.16	337.21	0.1221
25	5	5	1	5	1	3	1	3	5	44.20	336.17	0.1315
26	5	5	3	3	5	1	5	1	3	35.09	344.52	0.1019
27	5	5	5	1	3	5	3	5	1	39.50	343.48	0.1150

Table 2 **Optimized truss configurations from Taguchi analysis**

A	1	2	3	4	5	6	7	8	9
				(a) Average frequency, Hz, for truss member group					
1	35.80	36.71	36.61	35.54	37.45	34.51	**39.15**	36.32	35.69
3	37.56	37.70	37.64	37.85	**37.49**	38.10	37.02	**37.99**	37.97
5	**38.83**	**37.78**	**37.94**	**38.81**	37.25	**39.59**	36.02	37.88	**38.53**
				(b) Average mass, kg, for truss member group					
1	**318.43**	**320.52**	**323.65**	**313.21**	**314.25**	**316.34**	**306.95**	**307.99**	**310.08**
3	326.78	326.78	326.78	326.78	326.78	326.78	326.78	326.78	326.78
5	335.13	333.04	329.91	340.35	339.30	337.22	346.61	345.57	343.48
				(c) Average frequency-to-mass ratio, Hz/kg, for truss member group					
1	0.1147	0.1159	0.1143	0.1145	**0.1199**	0.1102	**0.1282**	**0.1193**	**0.1165**
3	0.1150	**0.1160**	**0.1162**	**0.1168**	0.1155	0.1173	0.1133	0.1162	0.1163
5	**0.1159**	0.1137	0.1152	0.1144	0.1102	**0.1181**	0.1041	0.1102	0.1128

Table 3 **Optimized and uniform truss performance**

Description	Configuration	f, Hz	m, kg	f/m, Hz/kg
Baseline uniform, $A = 3$	333 333 333	42.96	326.78	0.1315
Maximum frequency,	555 535 135	50.73	365.39	0.1388
Percent from baseline	——	+18.07	+11.82	+5.55
Minimum mass, $A = 1$	111 111 111	29.93	217.20	0.1378
Percent from baseline	——	−30.33	−33.53	+4.80
Maximum frequency-to-mass ratio	533 315 111	42.77	277.73	0.1540
Percent from baseline	——	−0.45	−15.01	+17.11
Uniform, $A = 5$	555 555 555	48.40	436.36	0.1109

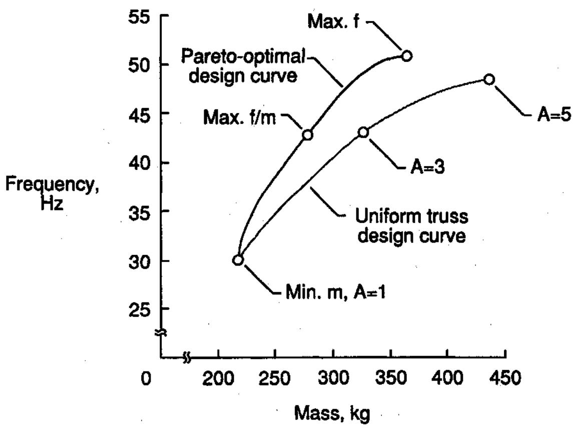


Fig. 5 Design curves for Pareto-optimal and uniform trusses.

The actual performance of each optimized configuration is determined from a vibrational analysis where the design variables are set to their predicted values in Table 2. For example, finite element analysis of the maximum-frequency configuration shows that this design has a fundamental frequency of 50.73 Hz and a mass of 365.39 kg. The three optimized truss configurations are compared with the baseline design (where every member group has a normalized area of 3) in Table 3. Also shown in Table 3 are results for the uniform truss where every member has the maximum normalized area of 5. The data from Table 3 are plotted in criterion space in Fig. 5.

Computation of Pareto-Optimal Design Curve

Low-order polynomial curve fits have been made to the optimized truss configurations in Table 3 to determine the Pareto-optimal design curve. The slope constraints discussed previously (zero slope at the maximum-frequency design, infinite slope at the minimum-mass design, and slope f/m at the design with maximum frequency-to-mass ratio) are applied by differentiation of the polynomial equations. A third-order polynomial with the general form

$$f = \alpha + \beta m + \gamma m^2 + \delta m^3 \tag{1}$$

is chosen to represent the design curve between the designs with maximum frequency and maximum frequency-to-mass ratio in criterion space. An equation for the slope in this region is given by differentiation of Eq. (1) with respect to m, resulting in

$$\frac{\mathrm{d}f}{\mathrm{d}m} = 0 + \beta + 2\gamma m + 3\delta m^2 \tag{2}$$

To determine the unknown polynomial coefficients α, β, γ, and δ, a matrix equation

$$\begin{bmatrix} 1 & m & m^2 & m^3 \\ 1 & m & m^2 & m^3 \\ 0 & 1 & 2m & 3m^2 \\ 0 & 1 & 2m & 3m^2 \end{bmatrix} \begin{Bmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{Bmatrix} = \begin{Bmatrix} f \\ f = f_{\max} \\ \mathrm{d}f/\mathrm{d}m = f/m \\ \mathrm{d}f/\mathrm{d}m = 0 \end{Bmatrix} \begin{matrix} \text{(max., } f/m \text{ design)} \\ \text{(max., } f \text{ design)} \\ \text{(slope at max., } f/m \text{ design)} \\ \text{(slope at max., } f \text{ design)} \end{matrix} \tag{3}$$

is assembled and evaluated with the program of Ref. 12. The rows of Eq. (3) correspond to the truss designs with maximum frequency-to-mass ratio and maximum frequency and the corresponding slopes. Since there are four unknown coefficients, an exact cubic fit is made through the two points in criterion space. Substitution of the appropriate values from Table 3 and solution of Eq. (3) gives

$$f = 45.6626 - 0.4520m + 0.002588m^2 - 0.000003593m^3 \tag{4}$$

Because the slope at the minimum-mass design is infinite, Eq. (1) cannot be used for the design curve between the designs with minimum mass and maximum frequency-to-mass ratio. However, the reciprocal $\mathrm{d}m/\mathrm{d}f$ of the slope is zero, which means that an equation of the form

$$m = \varepsilon + \zeta f + \eta f^2 + \theta f^3 \tag{5}$$

can be used to approximate the design curve between those designs. Equation (5) is solved for the polynomial coefficients ε, ζ, η, and θ as described previously, resulting in an equation

$$m = 1228.1303 - 83.4996f + 2.1941f^2 - 0.01780f^3 \tag{6}$$

which represents the Pareto-optimal design curve between the designs with minimum mass and maximum frequency-to-mass ratio.

Table 4 Optimized trusses from Taguchi and full-factorial analyses

Criterion	Configuration	f, Hz	m, kg	f/m, Hz/kg
Maximum frequency				
Taguchi analysis	555 535 135	50.73	365.39	0.1388
Full-factorial analysis	555 555 133	51.19	361.22	0.1417
Percent error	——	+0.90	−1.15	+2.05
Minimum mass				
Taguchi analysis	111 111 111	29.93	217.20	0.1378
Full-factorial analysis	111 111 111	29.93	217.20	0.1378
Percent error	——	0.00	0.00	0.00
Maximum frequency-to-mass ratio				
Taguchi analysis	533 315 111	42.77	277.73	0.1540
Full-factorial analysis	515 335 111	46.23	287.12	0.1610
Percent error	——	+7.48	+3.27	+4.35

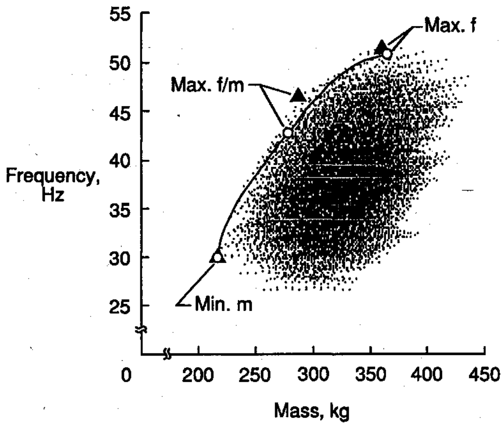


Fig. 6 Full-factorial data and Parteo-optimal design curve: o, Taguchi analysis; ▲, full-factorial analysis; and ——, Pareto-optimal design curve.

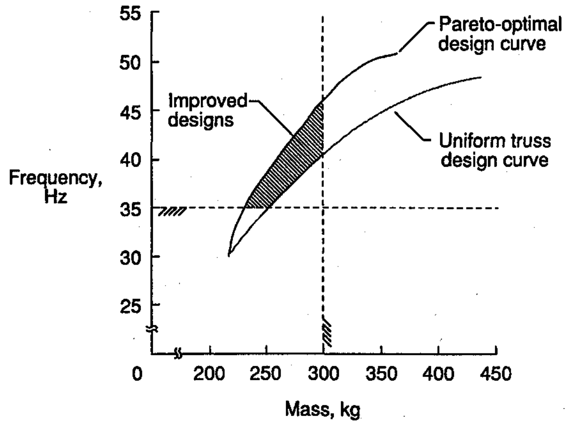


Fig. 7 Pareto-optimal and uniform-truss design curves with constraints.

The third-order curves defined by Eqs. (4) and (6) are shown in Fig. 5. This pair of curves is used to predict the tradeoff between frequency and mass for the Pareto-optimal truss designs.

Verification of Pareto-Optimal Design Curve

To determine the accuracy of the Pareto-optimal design curve, frequency and mass were computed for the 19,683 full-factorial cases. Criterion-space results of the full-factorial analyses are shown with the design curve in Fig. 6. Although the design curve shown is not the exact Pareto-optimal boundary for the full-factorial data, over 99.50% of the full-factorial data are bounded by the cubic curve fits from Eqs. (4) and (6). The trusses that have the highest frequency, lowest mass, and highest frequency-to-mass ratio of the 19,683 full-factorial configurations are extracted and compared with corresponding results from the Taguchi analysis in Table 4. The percentage error is the difference between the full-factorial and Taguchi values divided by the full-factorial value. Close agreement between frequency and mass values is observed for the maximum-frequency and minimum-mass truss designs. Results for the truss designs with maximum frequency-to-mass ratio show an error of 4.35% for the frequency-to-mass ratio.

Two key assumptions are necessary for the design method in this study to be valid. The first assumption is that a low-order polynomial is sufficient to define the Pareto-optimal design curve. Examination of Fig. 6 shows that, for this problem, a cubic polynomial curve provides a reasonable upper bound to the full-factorial data. However, a cubic Pareto-optimal boundary does not work for every design problem. The present method is applicable to two of the three analysis cases in Ref. 3, which have smooth, continuous Pareto-optimal boundaries. The boundary of the third case in Ref. 3 is highly discontinuous, and the present method would not provide a good approximation of the boundary.

The second assumption is that every point on the Pareto-optimal design curve has a corresponding feasible design. Although this second assumption may be true when the design variables vary continuously between limits (as in calculus-based optimization), it is not the case in this example, where the design variables take on only discrete values. Thus, although not every point on the design curve in Fig. 6 has an associated configuration in the full-factorial data, many feasible designs are very close to the design curve. Note that the design curve bounds the region of feasible designs, but does not tell the designer what the values of the design variables are for trusses along that boundary. However, detailed information about some configuration that meets the requirements is typically not required at a preliminary design level; instead, confirmation that an acceptable design exists is sufficient.

Truss Platform Preliminary Design Study

The first step in preliminary design is to determine the performance improvements that can be achieved through optimization. Such estimates can be made by comparing the performance of a uniform truss with that of the Pareto-optimal structures predicted by Eqs. (4) and (6). To provide a design curve for uniform trusses, a second-order curve fit [Eq. (1) with $d = 0$] is made to the uniform-truss frequency and mass data from Table 3. This curve fit is exact, since there are three uniform truss designs ($A = 1$, 3, and 5) and three unknowns. Solution for the polynomial coefficients gives

$$f = -18.3359 + 0.2909m - 0.0003162m^2 \tag{7}$$

The uniform-truss design curve from Eq. (7) is plotted in Fig. 5 with the Pareto-optimal design curve. Note that a semiempirical equation for the uniform-truss frequency is presented in Ref. 6, which may be used instead of Eq. (7). The (111 111 111) configuration is common to both the uniform and Pareto-optimal design curves, since it is both a uniform truss with $A = 1$ and the minimum-mass design.

The frequency and mass criteria, which have been considered unbounded until now, are typically constrained by operational requirements. To illustrate how the Pareto-optimal and uniform-truss design curves developed above are used in a practical design problem with constrained criteria, consider a 35-Hz lower bound on frequency and a 300-kg upper bound on mass as shown in Fig. 7. The shaded region labeled improved designs in Fig. 7 contains the truss

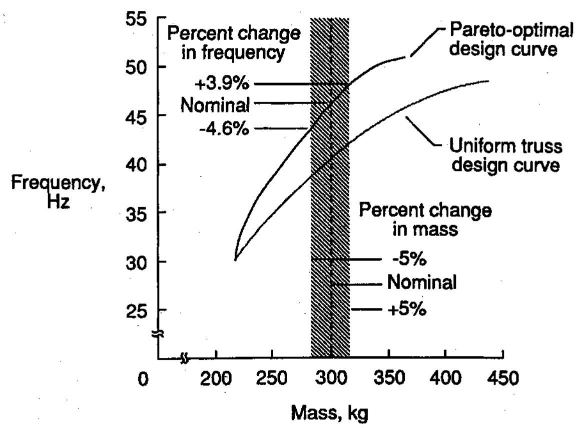


Fig. 8 Frequency sensitivity for mass-constraint variation.

designs that satisfy both the frequency and the mass constraint, and are bounded by the Pareto-optimal and uniform-truss design curves. From Fig. 7, a structural designer can immediately determine that both optimized and uniform truss designs are available that satisfy the given set of constraints. However, if the frequency constraint is increased to 50 Hz, it is clear from Fig. 7 that there are no optimized or uniform-truss designs that have frequencies above 50 Hz and mass less than 300 kg.

As part of the preliminary design process, the Pareto-optimal and uniform-truss design curves [Eqs. (4), (6), and (7)] can be used to estimate how much the mass can be decreased or the frequency increased through optimization. For example, assuming the constraints in Fig. 7, the fundamental frequency of an optimized truss with 300-kg mass will be 5.49 Hz (13.56%) higher than for a uniform-truss of equal mass. Likewise, the mass of an optimized truss with a fundamental frequency of 35 Hz is 22.59 kg (8.93%) lower than a uniform truss with equal frequency. These changes are the performance gains achievable from single-criterion optimization of the truss subject to the mass and frequency constraints.

In preliminary design problems, the constraint values are often ill defined, but nominal values must still be used for trade studies and performance estimates. The Pareto-optimal design curve may be used to predict the sensitivity of one criterion to variation in another criterion. For example, if the uncertainty in the mass constraint is $\pm 5\%$ (300 ± 15 kg), then Eq. (4) may be used to determine the resulting variation from the nominal 45.97-Hz frequency. For a 5% reduction in the mass constraint (to 285 kg), the frequency is reduced by 4.55% (to 43.88 Hz). An increase of 5% in the truss mass (to 315 kg) results in an increase in frequency of 3.92% (to 47.77 Hz). These results are shown graphically in Fig. 8.

One further aspect of multicriterion preliminary design must still be addressed: selection of a nominal design. That is, of the improved designs in Fig. 7, which one should be chosen for later studies? To answer this question, two additional considerations must be examined. First, the criteria must be prioritized in terms of their relative importance. In this truss design problem, is high frequency more important than low mass, or vice versa? Selection of one overriding criterion implies that the other criterion (or criteria) can be treated as a constraint. The original multicriterion problem now resembles a typical single-criterion problem, where one criterion is considered dominant, and all other considerations are handled with constraints in the optimization. The difference lies in the fact that in the present method, the constraints are not imposed until after the Pareto-optimal designs are located in criterion space, instead of being imposed prior to optimization. The second consideration requires estimation of the constraint variations from experience or historical data. For example, is 5% growth in the 300-kg mass constraint likely, or is 10% a more realistic value? What is the resulting variation in frequency? What is the expected variation in the 35-Hz frequency constraint, and the associated mass variation? Selection of a design whose behavior is least sensitive to variations in the constraints would be most prudent.

Concluding Remarks

An efficient method is presented for multicriterion preliminary design and demonstrated for a 14-m-diam, three-ring tetrahedral truss platform. This method permits rapid estimates of the truss natural frequency and mass, the criteria of interest, to be made during preliminary design studies. The method is based on construction of a Pareto-optimal design curve, generated from only 31 analyses (27 cases from the orthogonal array, plus verification of 2 optimized designs and 2 uniform trusses) of a possible 19,683 designs. The multicriterion design method presented here does not depend on which optimization technique is used to generate the optimal truss designs with respect to frequency, mass, and frequency-to-mass ratio; however, Taguchi methods are both accurate and efficient for this problem. The full-factorial data are used to demonstrate the validity of assumptions made on the nature of the Pareto-optimal boundary.

The present method does not identify what specific combination of design variables will return the values of the criteria. Instead, the method indicates that a configuration that satisfies the given constraints may exist, which is probably more important during a preliminary design phase than details of a specific configuration. The ability to estimate criterion sensitivity to variation of constraints is very important in preliminary design problems, when constraint values are often ill defined and may also change rapidly. Extension of this method to include additional criteria, such as cost, should be fairly straightforward, but physical insight may be lost for problems that have more than three criteria. The method discussed has been successfully applied to optimal design of a truss platform for maximum frequency and minimum mass, and may also be applicable to other design problems where conflicting criteria must be satisfied.

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